

Standard Deviation

Adapted by Anne F. Maben from "Statistics for the Social Sciences" by Vicki Sharp

Have you ever watched the final tournaments of Japanese Sumo Wrestling? The average weight of the contestants at the last tournament was, say, 315 pounds. Suppose a wrestler weighing 350 arrived at the last minute. What would he do to the average weight of the contestants? Clearly, he would raise it. Now, do you think every contestant would be saying to themselves, "Wow! Is that guy fat? Not likely.... OK, so how fat is this guy? Or, more to the point we'll be making, would we consider a weight of 350 somewhat distant from the mean of 315?

What if all the contestants in the tournament had weights clustered between 300 and 330? Then this late arrival would be the heaviest person there. On the other hand, if there were people who weighed 200, 300, 400, and 500 pounds, then his weight would be pretty close to the average. The big question, then, is how widely dispersed are the scores (or weights) about the mean? The dispersion measure statisticians use the most is the **standard deviation (SD)**.

Finding the standard deviation is as bad as it gets in statistics. You'll need to do a lot of calculations. But there's one bit of good news: You can do all those calculations on a pocket calculator! The formula may at first seem intimidating, but you'll get used to working with it and maybe even *like* it...

We're still not ready to define just what the standard deviation is, but you may be able to make a guess about its magnitude. Remember that SD measures dispersion of scores about the mean. So, if we have a large standard deviation, say, 40 pounds, what does that mean? It means that the weights are fairly widely dispersed. And if we have a relatively small standard deviation, say, 15 pounds? Then the weights are very closely clustered about the mean.

Here is an array of weights at the tournament: 297, 301, 306, 312, 314, 317, 325, 329, 334, and 350. Before we find the standard deviation, I'd like you to find the new mean for the numbers in this array. You should have gotten 318.5. So this latecomer raised the mean weight of the contestants from 315 to 318.5. How much is the standard deviation of weights at the tournament? To find out we follow a familiar three-step process:

1. **Write down a formula;**
2. **Substitute numbers into the formula; and**
3. **Solve for the unknown.**

Here's our formula.
$$\sqrt{\frac{N \sum X^2 - (\sum X)^2}{N(N-1)}}$$

We'll go over each of the terms here. The symbol that encloses everything else, is a square root sign. It means you need to find the square root of everything within the sign after you've done all the other stuff you're supposed to do. But don't worry; you'll be able to find the square root in about three seconds with your calculator. In case you don't remember the other terms, **N** is the number of values in a data set. The Greek letter sigma, or \sum , means "the sum of." Here we'll be summing the squares of each of the values in a data set. The next term, $(\sum X)^2$, is often misused. We sum up all the values, and then we square that sum. Got all that? Well, don't worry, because I'm going to work out this problem step by step.

X	X ²
297	88,209
301	90,601
306	93,636
312	97,344
314	98,596
317	100,489
325	105,625
329	108,241
334	111,556
350	122,500

$$\begin{aligned} \text{standard deviation} &= \sqrt{\frac{N \sum X^2 - (\sum X)^2}{N(N-1)}} = \sqrt{\frac{10(1,093,597) - (3185)^2}{10(10-1)}} \\ &= \sqrt{\frac{10,935,970 - 10,144,225}{10(9)}} = \sqrt{\frac{781,645}{90}} \\ &= \sqrt{8684.944} = 93.19 \end{aligned}$$

$\sum X = 3,185$ $\sum X^2 = 1,016,797$

We've gone ahead and added the X's. Now we're ready to substitute numbers into the standard deviation formula. First, how much is N? **N is 10** since there are 10 values in the data set. What about $(\sum X)^2$? How do we find that number? What do you think? We just square the sum of the X's, or multiply 3,185 x 3,185. That gives us 10,144,225. Finally, for the denominator, N(N - 1), we substitute 10(10 - 1).

Do you need to memorize this formula? **Not unless you're a masochist!** If you've got this paper, you'll be able to look up this formula. Remember, the trick is not to be able to memorize formulas, but to be able to use them. Besides, you'll be able to plug in the data and hit the "calculate standard deviation" key of your statistical calculator. To truly understand what "Standard Deviation" is and how we use it in everyday life, we first need to discuss the "normal curve."

The Normal Curve

On the first day of the semester, someone in the class can always be counted upon to ask, "Do you grade on a curve?" If I did, most people in class would be getting C's, not the A's or B's they deserved, since Poly's science students are so sharp that their grades do NOT fit a normal distribution. When we talk about grading on a curve, we are talking about the normal curve, often referred to as *the bell curve*. Why are they called bell curves? Take a look at the first figure and you tell me. To be fair, only the middle of the curve is shaped like a bell. The curve also has tails, extending to the left and the right. We need to see how the normal curve works before we can use it for grading or for any other purpose.

The Standard Normal Distribution

The mean is literally the central score in a normal distribution. In Figure 1, the mean, is smack in the middle of the curve. The normal curve is marked off by *standard deviations*. Since the mean is exactly at the center of the curve, it is "0" standard deviations away from the center. If we move one standard deviation to the left of the mean, we are -1 standard deviation from the mean. Similarly if we move one standard deviation to the right of the mean, we are +1 standard deviation from the mean. If we move farther to the right, we'll reach 2 standard deviations and 3 standard deviations. And if we move farther to the left from the mean, we would reach -1, -2, and -3 standard deviations. What does all of this mean?

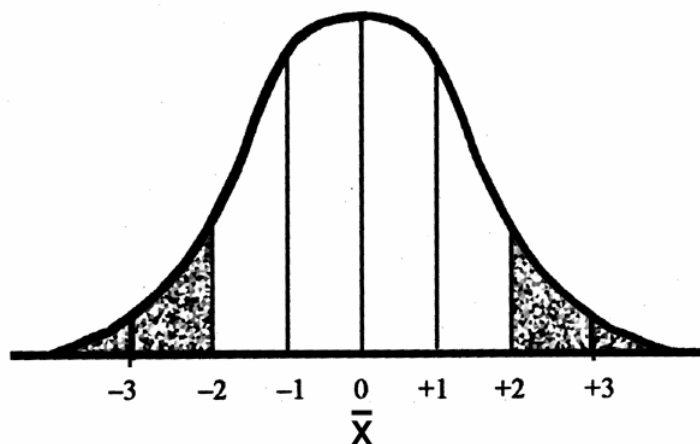


Figure 1.

I hope Figure 2 will help answer that question. You'll notice that the normal curve is divided into sections bounded by standard deviations. Each section is assigned a proportion, or percentage. For example, the section between -1 standard deviations and 0 standard deviations has a percentage of 34.13%. This means that in a normal distribution, 34.13% of the measurements, or observations, would occur between -1 standard deviations and 0 standard deviations. And 13.59% of all observations would be between -1 standard deviations and -2 standard deviations. Since 2.15% of the observations would lie between -2 standard deviations and -3 standard deviations, and 0.13% of all observations would lie to the left of -3 standard deviations, then what is the total percentage of observations that lie to the left of the mean?

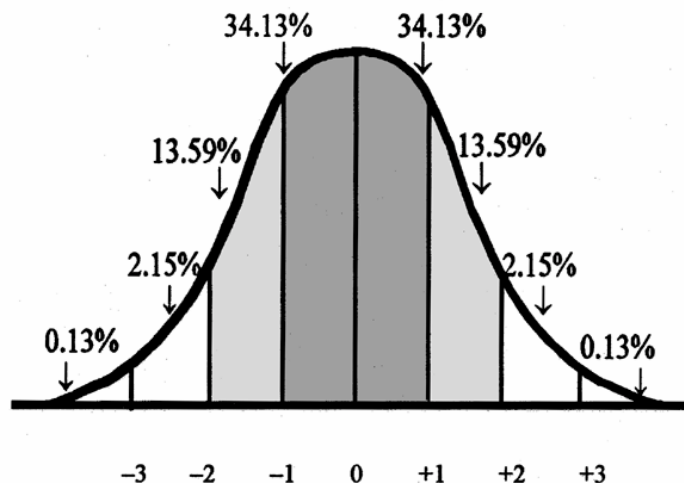


Figure 2.

Fifty percent of all observations lie to the left of the mean. And since the standard normal curve is symmetrical, the sections bordered by the positive standard deviations are identical to those on the left half of the curve. So 50% of the area under the curve is to the left of the mean, and 50% of the area under the curve is to the right of the mean.

Suppose we had a score, or observation, that was equal to the mean. What would be the percentile rank of that score? It would be in the 50th percentile. Ready for another one? Suppose someone got a really low score, say at exactly -2 standard deviations. See if you can figure out its percentile rank. It would be in the 2.28th percentile (0.

13% + 2.151/6). In other words, it would be higher than just 2.28 percent of the scores. And what percent of scores would it be lower than? It would be lower than 97.72 percent of all scores.

We've already said that the standard deviation is a measure of dispersion of scores about the mean. Let's be more specific. In a normal distribution, 68.26 percent of all scores will lie within one standard deviation of the mean; 95.34 percent of all scores will lie within two standard deviations of the mean; and 99.74 percent of all scores will lie within three standard deviations of the mean.

For purposes of this class, in analyzing data, *any calculated standard deviations that are **more than 2 standard deviations above or below the mean** will be considered unreliable*. They lie *outside* the 95% confidence limits for probability. Any deviations within the data did not occur from chance alone: something *was going on that affected the normal distribution of the data!*

This is not necessarily a bad thing to have happened. Say you are sampling the height of redwood trees in an old growth forest. From previous data, their height is known to be very uniform, with a mean of 260 ft. and a SD of 30 ft. After sampling, your data shows a mean of 210 ft. and a SD of 70 ft, more than 2 standard deviations from the mean. An observant scientist would start wondering what had occurred in the area that resulted in such a deviation from the normal distribution. Perhaps the El Niño of the previous winter had thinned out many tall trees and allowed young saplings to shoot up? SOMETHING must have been going on. Further research might provide an answer.

Also know that most populations in nature are NOT normally distributed! If we had a standard normal distribution, the mean would always be in the exact center of a symmetrical curve. Very often curves depicting this data tail off to the left or to the right. We term such curves **skewed**. The curve in figure 3 is negatively skewed because it tails off to the left and a few extreme low scores pulled down the mean. If the scientist plotted the heights of redwood trees in the old growth forest that was sampled, the curve would be skewed to the left after the El Niño, when compared with its usually normally-distributed heights. Teachers HOPE that grade distributions after tests will show a skew towards the "A" side of the curve, showing that students did significantly *better* on the test than expected. Curves of class test results that are high on both ends and show a drop in the middle say that students either knew the material or they didn't – not much in between. This is called an **inverse curve** and usually shows up in senior classes after college acceptances are in... What do you suppose might be happening to cause this effect?????

